ispPAC®10 Biquad Filter Implementation

Lattice Semiconductor Corporation

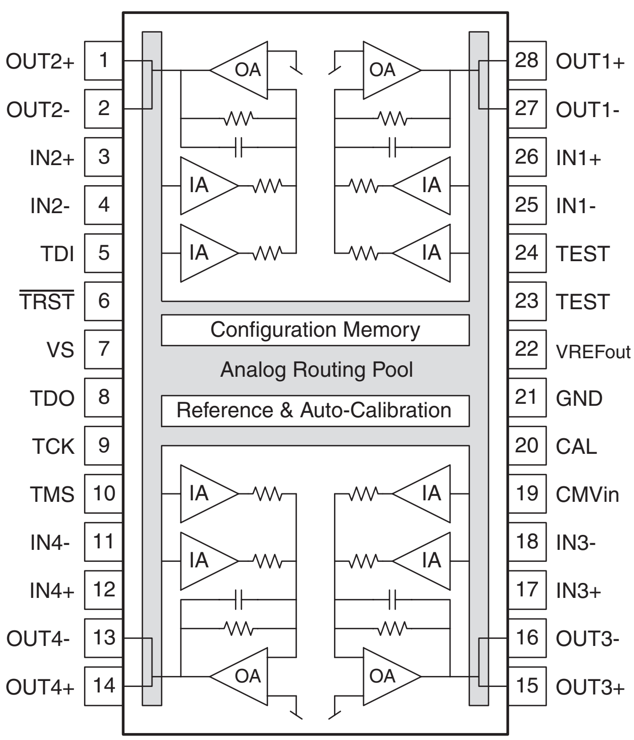
Introduction

This application note describes how to use the ispPAC10 to build precision biquadratic filters. In-system programmability (ISP™) enables programming, verification and reconfiguration directly on the printed circuit board using the IEEE standard 1149.1 compliant serial port. Second or fourth order continuous-time lowpass and bandpass filters in the range of 10kHz – 100kHz can easily be built using the ispPAC10.

The ispPAC10 contains four integrated programmable analog modules known as PACblocks and a programmable interconnect system (Figure 1). Each PACblock emulates a collection of op amps, resistors and capacitors. Because of the flexibility of the ispPAC10, a filter's gain, Q and cutoff frequency can be varied without the need for external components.

The next several pages develop the theory behind biquad filters. For those less interested in theory and more interested in practice, a simple tool for realizing biquad filters with the ispPAC10, included in PAC-Designer®, a Windows®-based design tool, is discussed in the section "PAC-Designer Macro."

Figure 1. ispPAC10 Block Diagram



Background

Filtering is a fundamental signal processing function: it may be performed prior to sampling with an ADC (an anti-alias filter), used to reduce noise bandwidth, enhance a range of frequencies, or for a variety of other reasons. A single PACblock can be configured as a programmable single pole lowpass filter or an integrator. Multiple PACblocks can be connected together inside one ispPAC10, permitting higher-order filters to be realized. There are many topologies for synthesizing more complicated filter transfer functions. One common approach is based upon dividing the transfer function into biquadratic sections:

$$H(s) = K\frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \quad (1)$$

Individual biquadratic sections can be cascaded to generate higher order filters. It is possible to dissect filter polynomials, including the Butterworth and Chebyshev polynomials, into biquadratic sections and synthesize each section. The cascaded biquad sections then generate the overall filter response. An advantage of cascading is that higher-order transfer functions are reduced to simpler second-order functions. Furthermore, the individual sections are isolated, making tuning of the filter characteristics straightforward. By manipulating the poles and zeros of the individual biquad sections, complex filters with reasonably low sensitivity to component tolerances can easily be realized.

Filter Parameters

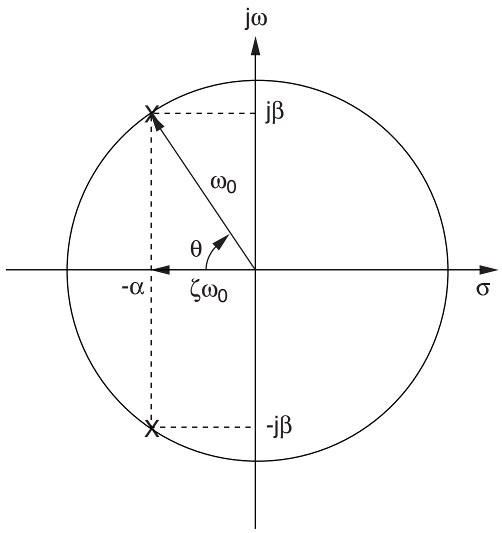
Filters are classified by the function they perform, usually in terms of their frequency characteristics, such as lowpass, bandpass, etc. In some cases, though, they perform a function best understood in the time domain, such as with delay equalizers.

The general expression for a second order function is commonly expressed in terms of the damping ratio ζ and undamped natural frequency ω_0.

$$H(s) = s^2 + 2\zeta\omega_0 s + \omega_0^2 \quad (2)$$

When s<1, the roots of this expression are complex conjugates. Figure 2 shows the roots in the s-Domain.

Figure 2. s-Domain



$$\omega_0 = \sqrt{\alpha^2 + \beta^2} \tag{3}$$

$$\theta = \cos^{-1}\zeta \tag{4}$$

An alternative expression for a second order system uses Q in place of the damping ratio:

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{\omega_0}{Q}s + \omega_0^2 \tag{5}$$

This is the form familiar to filter designers. The damping ratio and Q are related by:

$$Q = \frac{1}{2\zeta} \tag{6}$$

The poles of a transfer function are known as the natural poles of the system because they are not a function of the input waveform. When these roots (poles) become complex, the system response becomes underdamped. As the roots approach the jω axis (from the left), the effect is seen in both the time domain, as increased overshoot and settling time, and in the frequency domain as peaking in the magnitude response. This occurs because the roots moving closer to the jω axis corresponds to reduced damping ratio or higher Q. Biquad sections can be characterized by ω_0 and Q. By manipulating these values for individual biquad sections, an overall cascaded filter response can be achieved.

Biquad Topologies

There are many implementations of the basic biquadratic function of Equation 1. One common approach uses a single op amp per biquadratic. These single stage biquads are classified as either positive or negative feedback. This is a broad categorization and many realizations have been proposed. The topologies differ in realizability issues, including the number of components, component spread, suitability to high or low frequency poles, classes of filter functions provided and sensitivity to the active and passive components.

Three Amplifier Biquad

An alternative topology uses three amplifiers in the realization of the biquadratic function. This is appropriately known as the three amp biquad or state-variable biquad. The advantages of this topology include its versatility in the filter functions it provides, its ease of tuning and its good sensitivity performance. Ease of tuning and low sensitivity permit the implementation of moderate Q filters. The block diagram for the three amp biquad is shown in Figure 3.

The filter is composed of lossy integrators and integrators. The use of individual op amps to implement these functions is shown in Figure 4.

Figure 3. Three Amplifier Biquad Block Diagram

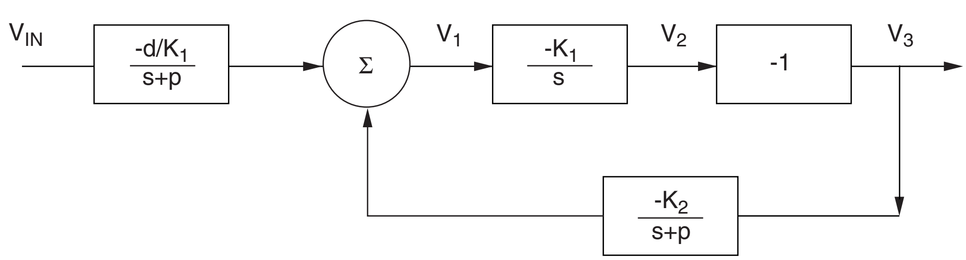


Figure 4. Three Amplifier Biquad Implementation

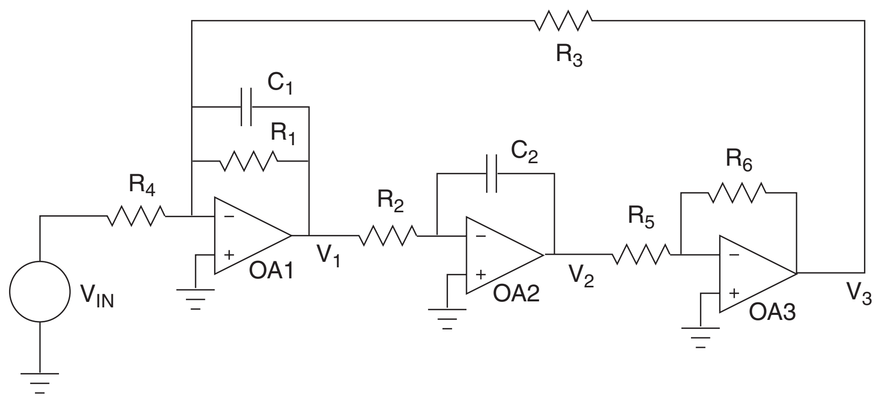
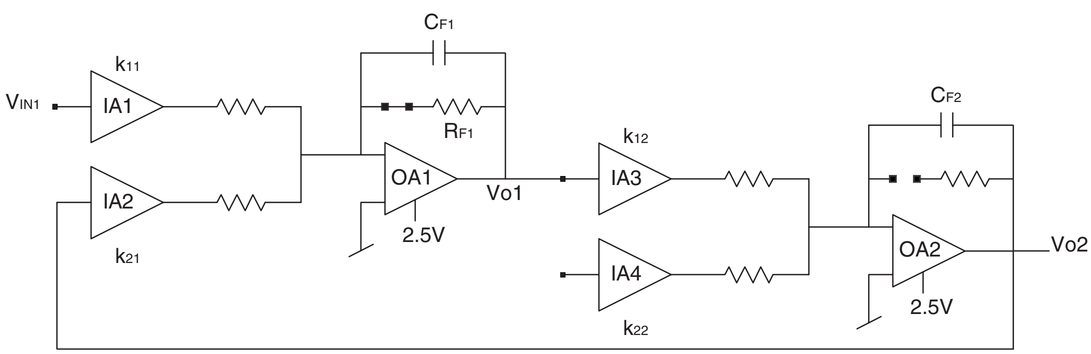


Figure 5a. ispPAC10 Implementation of the Three Amp Biquad Schematic



$$\frac{V_{O1}}{V_{IN1}} = \frac{\dfrac{k_{11}s}{C_{F1}\cdot 250k\Omega}}{s^2 + \dfrac{s}{(C_{F1}\cdot 250k\Omega)} - \dfrac{k_{12}k_{21}}{(C_{F1}\cdot 250k\Omega)(C_{F2}\cdot 250k\Omega)}} \quad \text{(bandpass)} \qquad (7)$$

$$\frac{V_{O2}}{V_{IN1}} = \frac{\dfrac{k_{11}k_{12}}{(C_{F1}\cdot 250k\Omega)(C_{F2}\cdot 250k\Omega)}}{s^2 + \dfrac{s}{(C_{F1}\cdot 250k\Omega)} - \dfrac{k_{12}k_{21}}{(C_{F1}\cdot 250k\Omega)(C_{F2}\cdot 250k\Omega)}} \quad \text{(lowpass)} \qquad (8)$$

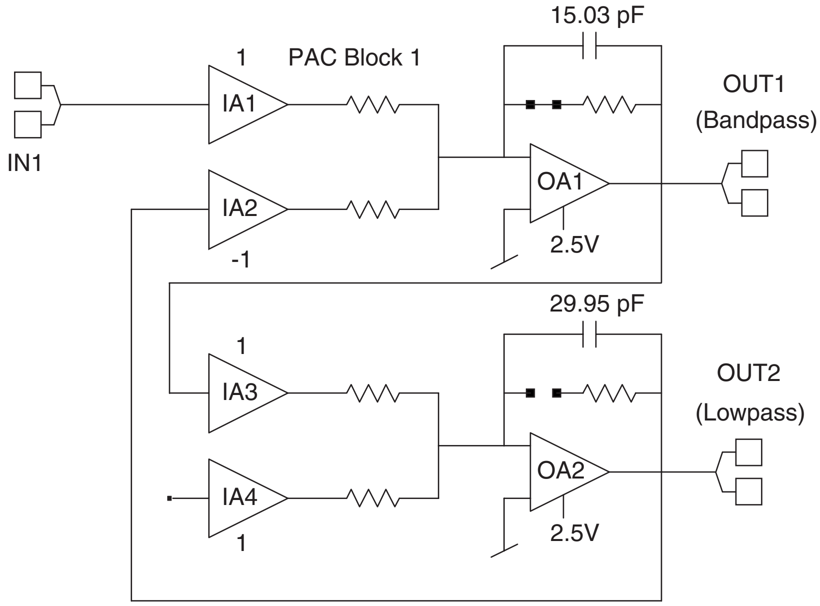
ispPAC10 Biquad

The ispPAC10 is capable of realizing each of the fundamental signal processing functions shown in Figure 3. In fact, because a PACblock can sum or difference two signals, the three amp biquad can be implemented using two PACblocks and the internal interconnect as shown in Figures 5a and 5b.

The transfer function for each output is shown in Equations 7 and 8. The k values are the programmed gains for each input amp and the C_F values are the feedback capacitor values in Figure 5a. (250kΩ is the equivalent single-ended resistance value for the input amp).

Tracing the signal from input to feedback shows that k_{21} needs to be negative to ensure negative feedback. This inversion is easily implemented by choosing a negative gain setting for IA2. The Biquad Filter macro automatically takes this into account, thereby avoiding the need for OA3 from the “discrete” implementation of Figure 4.

Figure 5b. ispPAC10 Three Amp Biquad Implementation



The general expressions for lowpass and bandpass filters are:

$$H(s) = \frac{Av_{DC} \cdot \omega_0^{\;2}}{s^2 + \frac{\omega_0}{Q} s + \omega_0^{\;2}} \qquad \text{(lowpass)} \qquad (9)$$

$$H(s) = \frac{Av_{BP} \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q_0} s + \omega_0^{\;2}} \qquad \text{(bandpass)} \qquad (10)$$

Comparing Equations 7, 8, 9 and 10 and defining $k_n = |k_{21}|$, the following expressions can be derived:

$$\omega_0 = \sqrt{\frac{k_{12}k_n}{(C_{F1} \cdot 250k\Omega)(C_{F2} \cdot 250k\Omega)}} \qquad \text{(lowpass)} \qquad (11)$$

$$Q = \sqrt{\frac{C_{F1}}{C_{F2}} k_{12}k_n} \qquad \text{(bandpass)} \qquad (12)$$

$$Av_{DC} = \frac{k_{11}}{k_n} = -\frac{k_{11}}{k_{21}} \qquad (13)$$

$$Av = |k_{11}| \qquad (14)$$

For practical ispPAC10 filters, the programmable capacitor array, with more than 120 pole locations and a typical pole accuracy of ±1.0%, is used to give f_0 values between 10kHz and 100kHz. The maximum PACblock gain of 10 limits the Q values of practical filters to ≤10. The maximum input amplitude, which is a function of filter gain and Q, needs to be limited to avoid overloading any of the stages within the PACblocks, but the programmable gain feature of the ispPAC10 makes this an easy task.

Focusing on the lowpass filter, for Q ≥ 1, there is peaking near f_0.[1] The amount of peaking is approximately Q, and

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[1]More precisely, for $Q > 1/\sqrt{2}$, $|LPF_{MAX}| = \frac{Av_{DC}}{\frac{1}{Q}\sqrt{1-\frac{1}{4Q^2}}}$ occurring at $\omega_{MAX} = \omega_0\sqrt{1-\frac{1}{2Q^2}}$ and for $Q < 1/\sqrt{2}$, the maximum is Av_{DC} occurring at DC.

in order for the feedback signal to not overload the k_{21} gain cell, the following limit applies for V_{IN1}:

$$V_{IN1} \le \left(\frac{6V_{PP}}{k_{21} \cdot Av_{DC}}\right)\left(\frac{1}{Q}\right) \qquad (15)$$

For Q < 1, the maximum amplitude is at DC, and again to not overload the k_{21} gain cell:

$$V_{IN1} \le \left(\frac{6V_{PP}}{k_{21} \cdot Av_{DC}}\right) \qquad (16)$$

Exceeding these limits introduces distortion as a PACblock will be driven out of its linear region of operation.

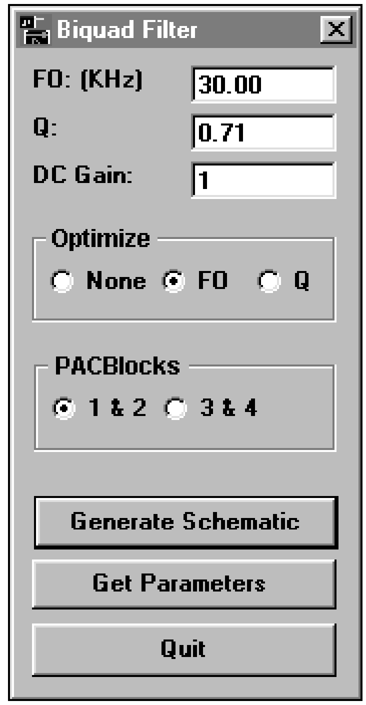
PAC-Designer Macro

Using Equations 11-14, a bandpass and/or lowpass filter can be designed. However, there is an easier way. Lattice's PAC-Designer software includes a macro that takes as its input the desired filter parameters and configures the PACblocks to closely achieve the requirements within the PACblocks' parametric range. Figure 6a shows the screen displayed when the Biquad Macro is run. Figure 6b shows the resulting circuit configuration from PAC-Designer. The PAC-Designer implementation uses one-half of the ispPAC10. PACblock1 is configured as a lossy integrator and the inversion occurs where OUT2 is summed with IN1 (IA2 of PACblock1).

This macro uses either the two left side or right side PACblocks to implement the filter. It also can extract the parameters from the schematic and update the user f_0, Q and gain values. The biquad filter that the macro implements is more conservative than that given in Equations 11 and 12. It does not manipulate k_{12} or k_{21}; only the capacitor values and k_{11} are used. This is more in line with keeping the Q value below 10 and, from Equations 15 and 16, makes it less likely that the feedback input k_{21} will be overdriven.

Because the ispPAC10 has integer steps in gain and stepped capacitor values, it may not be possible to build a filter with the exact f_0 and Q requested. In this case, the macro provides a best fit compromise and designs a filter with an f_0 and Q close to those entered. The actual filter values are then shown in their respective dialog boxes. Notice that the macro provides for optimizing either Q or f_0. When optimizing for f_0 (Q), the macro designs the filter to minimize the error in f_0 (Q) and the resulting Q (f_0) is of secondary concern. Again, the macro displays the actual f_0 and Q for the filter it has designed. With more than 120 pole frequencies between 10kHz and 100kHz to choose from, PAC-Designer generally finds an implementation within 3% of the desired f_0 or Q. By leaving the Biquad Filter macro window open, it is easy to try several filter variations to find the one that best meets your requirements. Note that in these filters, V_{01} is the bandpass response and V_{02} is the lowpass response.

Figure 6a. PAC-Designer Biquad Macro Entry Screen



Simulation and Results

PAC-Designer contains a gain/phase simulator that can display up to four transfer functions. (The simulator is accessible through the Tools menu or the simulator icon in the toolbar). Figure 7 shows the simulation results on this filter while Figures 8 and 9 are actual measurements taken with a network analyzer. Notice that in the actual measurements, the noise floor and the presence of excess phase at high frequencies limit the measurement.

Additional Application Information

To create filters of third or fourth order, the output of the left side biquad can be cascaded with a first or second order filter on the right side. Note that only OUT2 and OUT4 can be used by all four PACblocks. A band reject filter can be implemented by subtracting the bandpass filter from the input in a third PACblock. It will have a frequency response that is the inverse of the bandpass filter.

Figure 6b. PAC-Designer Fully-Implemented Three Amplifer Biquad

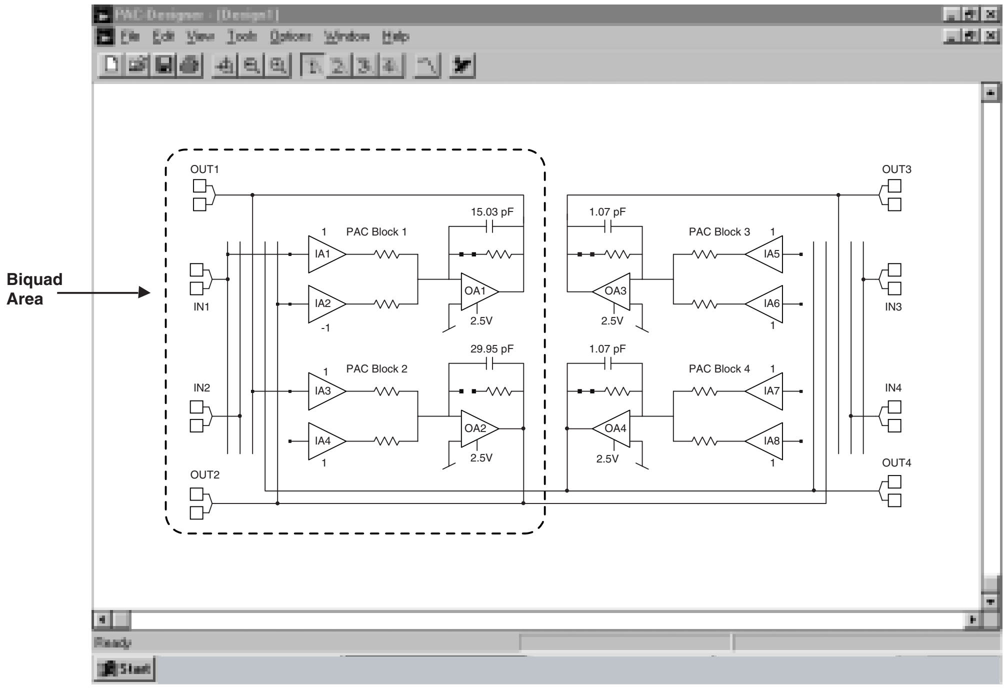
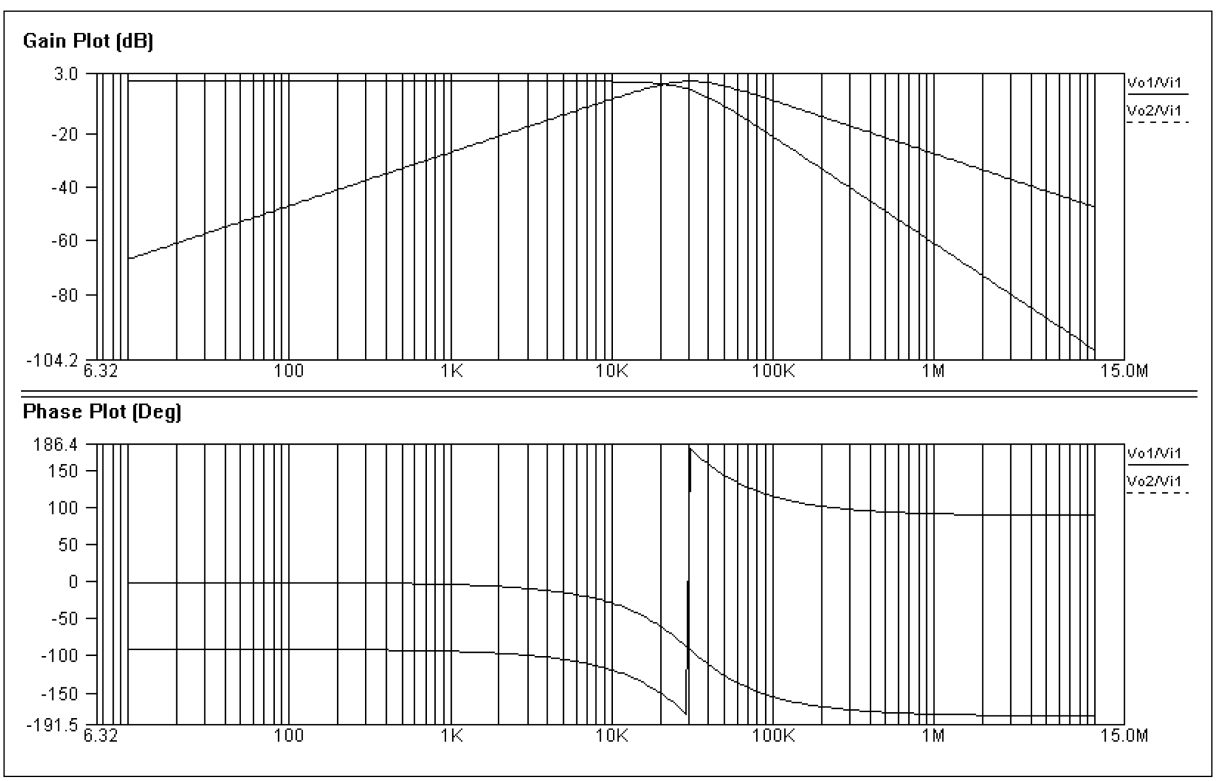


Figure 7. PAC-Designer Lowpass and Bandpass Simulation Screen Shot



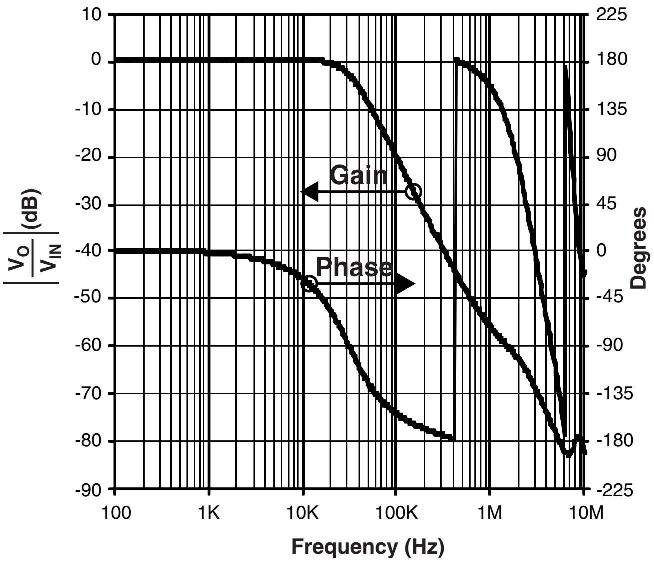


Figure 8. ispPAC10 Lowpass Measurement

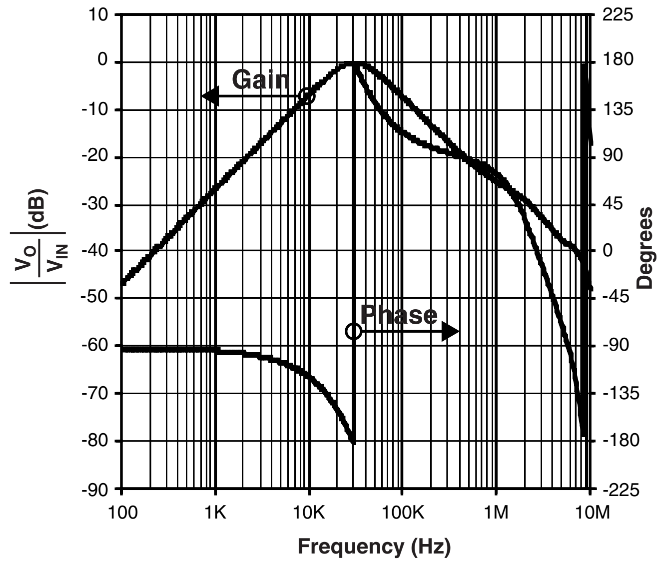


Figure 9. ispPAC10 Bandpass Measurement

Summary

Second or fourth order continuous-time precision lowpass and bandpass filters in the range of 10kHz–100kHz can easily be built using the ispPAC10. PAC-Designer software includes a convenient macro function which synthesizes a biquadratic filter from user-supplied requirements and its built-in simulator allows easy verification of filter performance. The ispPAC10 is ideally suited for building precision biquad filters because of its tightly-trimmed capacitor arrays and its high-impedance differential inputs. In-system programmability allows for easy programming and re-programming of filter parameters, even after the device has been soldered onto a circuit board, with no need for external component changes.

Technical Support Assistance

Toll Free Hotline: 1-800-LATTICE (Domestic)

International: 1-408-826-6002

E-mail: ispPACs@latticesemi.com

Internet: http://www.latticesemi.com